Relational word problems: A cross-cultural comparison

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Students' difficulty with mathematical word problems such as the 'students and professors' problem has been studied in mathematics education research. The difficulty may lie with the language or with choosing an appropriate schema. Translating these word problems into a language such as Chinese creates a new avenue of research, since the syntax of Chinese is different from the English version. This paper will discuss the results of part of a cross-cultural research masters thesis investigating problem processing of mathematical word problems in Chinese and English. The results suggest that while language appears to be a factor in the processing of relational word problems, the selecting of an appropriate schematic model is also an important factor.

Research has established that the cognitive effort required to process certain mathematical word problems is high in English (Clement, Lockhead & Monk 1981; Cooper & Sweller 1987; Lewis & Mayer 1987; and MacGregor 1991). Research in other languages on mathematical word problems has occurred in Europe (for example, Malle 1993), but it seems little has been done in Asian languages. Cognitive effort may not be so high in languages with different linguistic attributes which may allow for more efficient processing (than English) in certain circumstances.

Cross cultural language comparison between English and Chinese is interesting, particularly in mathematics, because, while the mathematics content is the same, there are notable differences in orthography and syntax. These characteristics of written language have been studied by researchers in cognitive psychology and psycholinguistics to determine the effects of cognitive processing in ordinary prose (Chen & Tseng 1992). In general, Flores d'Arcais (1992) has argued that the available evidence does not seem to indicate dramatic processing differences for words written in alphabetic or logographic scripts. However, Hoosain (1991) has argued that in a particular context and under certain circumstances cognitive advantage of Chinese over English is evident.

In the specific context of mathematics, the reading of sentences is a complex process. Students may read mathematical word problems and understand the sentences but have little idea how to solve the problem. Recent research on the problem solving process has involved two theories. Schema theory (Sweller 1993) associates the problem solving process with the semantic structure of the word problems. Dual coding theory looks at cognition as having two interconnected systems - one an imagery system and one a language system (Dawe & Anderson 1993). In the solving process, whichever theory is used, there appears to be a complex interaction between the words and the syntax, as well as the phonology, since, when solving many word problems, students often vocalise and subvocalise to help gain meaning. This interaction between the words, syntax and phonology may help to access the appropriate schema, building up an image to solve the problem. Many characteristics of a language thus assist in the access to problem processing in mathematics. However, in this paper, differences in syntax which may show differences in processing will be highlighted.

Many of the difficulties students encounter with mathematical word problems are those concerned with relational statements. Relational statements include transitive inferencing problems (sometimes called three term series problems or linear syllogisms) For example, *Ben is taller than George; Dave is shorter than George. Who is tallest?* Here there are two premises which are used to describe a linear ordering of the three terms (Ben, George and Dave). Each premise is converted into a two term array (eg B>G and D<G); then the two arrays must be integrated into a single three term array (eg B>G>D). This integration into the single array, according to Maybery, Bain and Halford (1986) imposes the greatest cognitive load. Sweller (1993) believes this to be intrinsic cognitive load since the load does not vary with restructuring. While we cannot restructure the sentences much within English, the restructuring carried out when a translation to Chinese is undertaken may give a different cognitive load.

Relational statements are also included in compare problems such as: *There are five more women than men. There are 25 women, how many men are there?* The difficulty with these relational statements has been studied by a number of researchers in America, Australia and Germany (see for example Clement, Lochhead & Monk 1981; Lewis & Mayer 1987; Scholnick 1988; MacGregor 1991; MacGregor & Stacey 1993; Malle 1993; Thomas 1994). The most commonly accepted reason for reversal error was due to the syntactic translation of the problem, but MacGregor & Stacey (1993), Thomas (1994) and Malle (1993) claimed there are other factors involved as well. As problems become more difficult reversal error becomes more evident, suggesting there are two processes occurring, one related to language, the other related to conceptual understanding.

Research by MacGregor & Stacey (1993) on 281 secondary school students required students to formulate equations from problems which lend themselves to correct syntactic translation. However, their results showed a high rate of error. For the question *z is equal to the sum of 3 and y. Write this information in mathematical symbols*, only 57% of students were correct. For the question *s and t are numbers. s is eight more than t. Write and equation showing the relation between s and t*, only 27% of students were correct. They concluded that there was no support for the belief that syntactic translation "is a frequently used method for formulating equations and a fundamental cause of errors". They claimed that the students' reversal errors are due to their attempts to construct mental models of the problem. These mental models do not necessarily correspond with the conventional mathematical code (for example in the phrase *s is 8 more than t* students may associate the 8 with the number *s* since *s* is the bigger number). Malle (1993) also claimed the reasons for errors to be quite complex and that syntactic translation is only used for the simplest of text. His views reflect the idea of MacGregor and Stacey (1993) that the mental model the student constructs does not match the formal mathematical conventional model. So while studies in linguistics suggest that English students use word order as a cue to sentence processing and that students process as they read, in mathematical word problems, students, while trying to make sense of the problem, produce a cognitive model which may or may not match the original linguistic model. When students incorrectly write $6m = w$ for *there are six times as many men as women*, the equal sign may mean correspondence, not equivalence (Malle calls this an "Entsprechungsschema"). With this equivalence model, students, who are then asked to find the number of men, given the number of women, should be able to find the correct answer as long as they do not use the conventional mathematical algorithms, since the internal code of the student does not match the external mathematical one. The results from these studies suggest that the problem may not be purely a language or conceptual problem but a combination of both.

If, in solving the problem, the construction of a mental model is more important than the language, then this problem should be evident in both Chinese and English speaking students although the Chinese may have the advantage if they have better developed visual skills. There may also be differences due to the more practised mathematical skills of the Chinese students.

If however, in solving the problem, language is an important factor then the Chinese[1] version of problems involving reversal error may provide some clues as to the nature of the language problem since the Chinese syntax in the problem is noticeably different from the English version.

One major difference between English and Chinese is the structure of sentences and the status of the verb. Sentences which include verbs are the dominant sentence type in Chinese language (Liuxun & Liu 1993). However there are many common sentences in Chinese where the verb is omitted and the predicate consists of a noun, adjective, adverb or phrase. For example: an English sentence: *Ben is taller than George*, is translated into Chinese as*: Ben bei George gaode*[2] *(Ben compare George tall)*. Here the word *tall* is classified as a stative verb. This type of construction, which is common in Chinese when using comparisons, has implications for processing. The English reader is alerted to **Ben** then the relationship, ie **height**, with the suffix "er" telling the reader a comparison is being made. In the Chinese version *Ben compare George tall*, the Chinese is alerted to **Ben** then the **comparison** of the two people then the relationship of **height**. The formation of a cognitive image in English is different from that in Chinese, presuming the reading continues from left to right in both languages. The English version creates an image of the tallness of Ben, while the Chinese version creates an image of the two people first. This difference may have important consequences for the development of an appropriate cognitive image in mathematical relational word problems.

The Chinese form of problems where reversal error occurs, tends to be more regular in the construction of these particular problem types. Chinese format in the following way:

Shuzi y shi shuzi x de 8 cheng

Number y is number x's 8 times (the number y is 8 times the number x)

which allows for correct and direct syntactic translation: $y = x8$. Moreover, the use of the word *de* is an indicator of possession and could be seen as a further clue to the positioning of the 8 with the x. However the syntax changes slightly in Chinese for addition /subtraction word problems:

nu huiyuan bi nan huiyuan duo wu

...female members compare male members more five (there are five more women than men)

In this question there is no verb but the adverb is used as a stative verb *duo* (more). (This construction is the same as the construction used in transitive inferencing questions such as *Ben compare George tall. (Ben is taller than George....)*. Here the adverb *tall* is always associated with the first noun.) While at first glance the construction allows for direct syntactic translation: $f = m + 5$, the construction of this type of compare statement may incorrectly associate the five with the female members giving an equation: $f + 5 = m$. The error may be in the correspondence model mentioned earlier as a possible model used in English. While there are a number of other differences between English and Chinese language, a study of the syntactical difference outlined above may give clues to the cognitive processes involved in solving mathematical word problems.

The purpose of the following experiment was to determine whether syntactic differences in relational statements between English and Chinese translate into differences in students' ability to solve these problems.

The Study

Subjects

Fifteen students from Taiwan who were enrolled in a preparatory program were selected for the study. The students from Taiwan had a mean age of 21.6 years (range from 17 - 32). They were preparing for entry into a business or commerce degree. Seven students had completed senior high school in Taiwan and their range of marks for mathematics was between 45% and 85%. Five students had completed high school at a different college with scores in mathematics ranging from 40% to 60%. Three other students were completing their studies in Australia.

All the 15 Australian students' first language was English and were in their first year of a commerce or business degree. The mean age was 19.4 years (range from 17 to 29). Seven students had completed Maths B at school with at least a sound achievement Another seven students completed the equivalent of Maths in Society with a range from low achievement to high achievement. One student had completed school in Malaysia but English was her first language.

Instruments

Two tests were used in this experiment and concentrated on the relational aspects of word problems. Test 1 consisted of ten context questions. Test 2 consisted of ten context free questions to match Test 1. The context free test was administered second so that it would not influence the results of the first test.

The questions were then translated into Mandarin (using traditional script) and input into a Macintosh computer by a Chinese translator. The authenticity and accuracy of the printout was verified by a research fellow at the university of Taiwan who sets mathematics questions for school students. The final translations were then checked by Taiwanese students at the University of Southern Queensland to ensure clarity. To test clarity of translation, the initial translation of the test was also sent to an institute in China where the test was trialed, using a pen and paper test, on a group of 16 second year students in a business college. In this case, the students were given all the questions and were thus able to change their answers after finishing the test.

Students were asked to read the questions on the computer screen and respond on the computer. A trial test of 5 questions was initially given to the students to ensure they understood the instructions and procedure. Then the two tests were administered. For both tests, one question at a time was displayed on the screen with students pressing the return button when they wanted to go on to the next question. Students were given pencil and paper for any working. During the test, the writer observed the students and at the end of the testing, students were interviewed.

Results and Discussion

Full results of the experiment can be found in Galligan (1997). The results in Table 1 relate directly to relational word problems used - five from Test 1 and two from Test 2. In the experiment there were many errors in both cohorts (75% for the Australian and 68% for the Taiwanese). While the Chinese structure of these sentences is more regular this apparent syntactical aid did not appear to assist Taiwanese students' overall results. The following examples illustrate this.

In question 2.3 (*y is eight times z*) using MacGregor and Stacey's definition of syntactic translation (1993), six Taiwanese and four Australian students used this method. All of the Taiwanese syntactic translations were correct while only two of the Australian responses were correct. This also substantiates earlier experiments where no Australian student used syntactic translation (Galligan 1997). From the results of these experiments, it appears that Taiwanese students use syntactic translation more than the Australian students.

In question 2.8 (*x is five less than y*) the Australian version does not translate syntactically directly but the Chinese version does. While the numbers correct are the same (five for both in second experiment and four for both in an earlier experiment (Galligan 1997)), ten Taiwanese students and six Australian students had reversal error. Only four Taiwanese students used a syntactic translation ($x = y - 5$: all correct). However, instead of using a syntactic model the Chinese version may have prompted the compare model used in transitive inferencing.

The transitive inferencing question: *Ben compare George tall* has the same syntactic structure as *x compare y small 5*. In the former case the word *tall* is linked to *Ben*. Using this compare model would correctly link the *small 5* to the *x*. The seven students' error occurring when putting the two variables into an equivalence statement ($x - 5 = y$).

In question 1.4 (*6 times as many shirts as trousers*), where correct syntactic translation was available for the Taiwanese but not for the Australians, only four of the Taiwanese correctly translated syntactically. The structure of this question is similar to question 2.3. It is interesting to note that 11 of the 13 mainland Chinese trial group responses chose this syntactic method to answer this question.

The result for question 1.4 is reflected in 1.7 (*eight more TVs than computers*) for the Australian students (3:3), but not for the Taiwnaese students (7:3). With question 1.7 only three Taiwanese students were correct. The Chinese structure of the sentence reflects question 2.8 where the preposition *bi* (compare) is used. However only 2 Taiwanese students had the incorrect solution $x + 8 = y$ and a further 2 students had $8x = y$. These two solutions could again suggest the linking with the model as mentioned in question 2.8. In this question the majority of errors occurred because of misreading of the question.

Table 1: Comparison of Australian and Taiwanese correct responses

	Australian (n=15)	Taiwanese (n=15)
Question	**Number correct**	**Number correct**
1.2 Ben is taller than George; Dave is shorter than George - Who is the tallest?	12	14
1.4 There are 6 times as many shirts as trousers in this shop. Use X for the number of shirts and Y for the number of trousers.	3	7
1.7 In a shop there are eight more T.V.'s than computers. Use x for the number of T.V.'s and y for the number of computers.	3	3
1.9 John is older than Fred. John is younger than Ken. Who is the oldest?	14	12
1.10 In a recipe for every cup of flour you use one third of a cup of sugar. Use x for the amount of flour and y for the amount of sugar.	5	0
2.3 Write this information in mathematical symbols: The number y is 8 times the number x	5	9
2.8 Write this information in mathematical symbols: The number x is five less than the number y	5	5

Question 1.10 (*for ever cup of flour you use 1/3 cup of sugar...*) produced different results. The syntactic pattern in Chinese in the previous questions (X compare Y adverb) was not seen in this question *(every use 1 cup flour then use three parts of one cups of sugar)*. Of the 15 Taiwanese students who were wrong in this question, 14 had reversal error. Of the 10 Australian students who were wrong, six had reversal error. There is something in the nature of this question when written in Chinese, which caused particular difficulties for the Chinese students. In Chinese, the syntactic and semantic structure of the question itself is different from the other relational word problems so this question does not easily fit into the schematic model used for the other problems. In the English version, the semantic structure is a little different (using the phrase *for every..)* but the syntactic structure is much the same as the other questions in which direct syntactic structure cannot be used. The other difference between the two translations is the different way the Chinese construct fractions. While Bell (1993) suggests conceptual advantage if fractions are expressed stating the denominator first, no advantage was found in the context of this experiment. The difficulty may lie solely with the language structure of the problem itself, the fraction component being irrelevant. However, it is a factor that cannot yet be ignored.

Structuring the language to allow for syntactic translation as in most of the Chinese questions, may appear to assist in the processing of these relational problems where syntactic translation is very clear as in question 2.3 and 1.4. However, it does not appear to aid processing where the translation is not as transparent. This reflects Malle's argument (1993) where he suggested syntactic translation is used only for the simplest of text.

The simple syntactic translation occurs in all the multiplication questions in Chinese but not in English. However this is not true for the addition questions. In the questions that involve multiplication, the Chinese had the verb *shi (is),* between the two things being compared (*shirts is trousers 6 times).* However, with the *more than* and *less than* statements the word between the two things was *bi (compare)* which is a preposition and the adverb such as *duo (more)* acts as the verb (*computers compare*

TV's more 8). For the 15 Taiwanese students in the two multiplication questions, there were 16 of the 30 questions correct, but for the two addition/subtraction questions only 8 of the 30 were correct. There was no such apparent difference with the Australian cohort (8 correct for both). The difference could be due to the different verb used - the *shi* prompting the correct model. The different cues used in sentence interpretation (ie the pre-verbal positioning to identify the subject in English and the post verbal cue to identify the object in Chinese) may allow Chinese readers to more correctly interpret the multiplication questions but a more extensive study would have to be undertaken to examine this more closely. Perhaps with an easy syntactic translation, Chinese may be able to pick up the model more easily than the English. There is also evidence that the Taiwanese students attempted to formulate equations more often than the Australian group (85% compared to 61%). Either the students have read the questions more carefully and hence knew they had to write an equation, or they are more expert in maths in that the word *fang cheng shi (equation)* prompted a response such as $x = 6y$ more often than their Australian counterparts. In their attempts to write more equations, the Taiwanese cohort may have used reversal error more often. But as other variables intervene, such as change in language, and order of presentation of questions, then similar errors occur in both languages. Moreover, when the question becomes more dissociated from a known model, error rates increase. Question 1.10 in the Chinese version is further from the model than the English version.

Conclusion

While the structure of relational word problems in written Chinese appears to advantage Chinese readers, in the experiments this advantage was not evident, providing further support for the suggestion of an appropriate cognitive model as the main cause of student error in relational word problems. The implication for educators may be an increased emphasis on developing in students an appropriate cognitive model to solve problems, rather than concentrating on the language difficulties of the problems themselves.

This study also provides a greater awareness of the difficulties and dangers in translating mathematical word problems and comparing results from different language groups. Just getting the language right does not necessarily mean the content conveyed by the language is the same. In cross - cultural mathematics tests, such as the recent Third International Mathematics and Science Study, this needs to be taken into consideration.

Notes

1. The Chinese language described here refers to Mandarin, the official language of mainland China and the the language used in the experiments was traditional Mandarin, the language used in Taiwan.
2. Full Chinese script version and tone marks for pinyin are not included here but are available from the author.

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